Printed Pages – 7

328411 (14)

BE (4th Semester)

Examination, Nov-Dec 2021

Branch : AEI, EI, Et & T, Mechatronics

MATHEMATICS IV

Time Allowed : Three Hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : Part (a) of each question is compulsory. Attempt any two part from (b), (c) and (d). Part (a) carries 2 marks. Part (b), (c) and (d) carry 7 marks each.

Q. 1. (a) Define Bessel's function.

(b) Solve in series the equation :

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$$

(c) Prove that :

$$J_{5/2}(x)=\sqrt{\frac{2}{\pi x}}\left\{\frac{3-x^2}{x^2}\sin x-\frac{3}{x}\cos x\right\}$$

(d) Prove that :

$$(2n+1)P_n(x)=P'_{n+1}(x)-P'_{n-1}(x)$$

Q. 2. (a) State and explain linear partial differential equation.

(b) Solve :

$$(mz-ny)\frac{\partial z}{\partial x}+(nx-\ell z)\frac{\partial z}{\partial y}=\ell y-mx$$

(c) Solve :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x\, \partial y} = \cos x \cos 2y$$

(d) Solve :

$$\left(D^2 + 2DD' + D'^2 - 2D - 2D'\right) z = \sin(x + 2y)$$

Q. 3. (a) Explain one dimensional heat flow.

(b) A string a stretched and fastened to two points ℓ apart motion is started by displacing the string in the form $y = a \sin(\pi x/\ell)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by :

$$y(x, t) = a \sin(\pi x/\ell) \cos(\pi x t/\ell)$$

(c) An insulated rod of length ℓ has its end A and B maintained at 0°C and 100°C respectively until steady state conditions prevails. If B is suddenly reduced to 0°C and maintained at 0°C. Find the temperature at a distance x from A at time t.

(d) A tightly stretched string with fixed end points $x = 0$ and $x = \ell$ is initially in position given by $y = y_0 \sin^3 (\pi x/\ell)$ if it is released from rest from this position. Find the displacement $y(x, t)$.

Q. 4. (a) Define Z-transform.

(b) Find Z-transform of the following :

(i) ℓ^{-an}

(ii) n sin xa

(c) Find the inverse Z-transform of :

(i) $\dfrac{2z^2 + 3z}{(z+2)(z-4)}$

(ii) $\dfrac{5z}{(2-z)(3z-1)}$

(d) Using Z-transform

Solve $y_{k+2} + 6y_{k+1} + 9y_k = 2^k$

given that $y_0 = y_1 = 0$

Q. 5. (a) Define random variable.

(b) A and B throw alternately with a pair of dice. A wins if he throws 6 before B throw 7 and B wins if he throws 7 before A throws 6. If A begins. Find his chance of winning.

(c) The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If two such pens are manufactured. Find the probability that none will be defective.

(d) Fit a Poisson distribution to the following data and test for its goodness of fit at level of significance .05

x :	0	1	2	3	4
f :	419	352	154	56	19